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Faster and more robust maximum likelihood estimation for random utility models

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Faster and more robust MLE for RUMs

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$$\rightarrow \begin{bmatrix} U_1 \\ U_2 \\ U_3 \end{bmatrix} = \boldsymbol{U}$$

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Faster and more robust MLE for RUMs

$$\rightarrow \quad \begin{bmatrix} U_1 \\ U_2 \\ U_3 \end{bmatrix} = \boldsymbol{U} = \boldsymbol{\mu} + \boldsymbol{X}(\beta + \gamma) + \varepsilon$$

Faster and more robust MLE for RUMs

$$\rightarrow \begin{bmatrix} U_1 \\ U_2 \\ U_3 \end{bmatrix} = \boldsymbol{U} = \mu + \boldsymbol{X}(\beta + \gamma) + \varepsilon$$

- μ are alternative-specific constants

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- β encodes how $\boldsymbol{X}$ influences $\boldsymbol{U}$
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Let $\boldsymbol{\theta}$ denote the set of model parameters and assume a distribution for ε is fixed.

1. calculate choice probabilities $P_{nj}(\boldsymbol{X}_n, \boldsymbol{\theta})$ for each decider n and alternative j
2. build log-likelihood function $\log \mathcal{L}(\theta \mid \boldsymbol{X}, y) = \sum_{n,j} 1(y_n = j) \log P_{nj}(\boldsymbol{X}_n, \theta)$
3. maximize $\log \mathcal{L}(\theta \mid \boldsymbol{X}, y)$ with respect to θ

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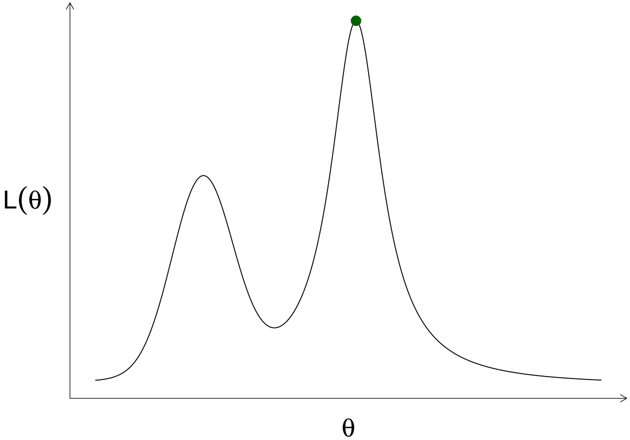

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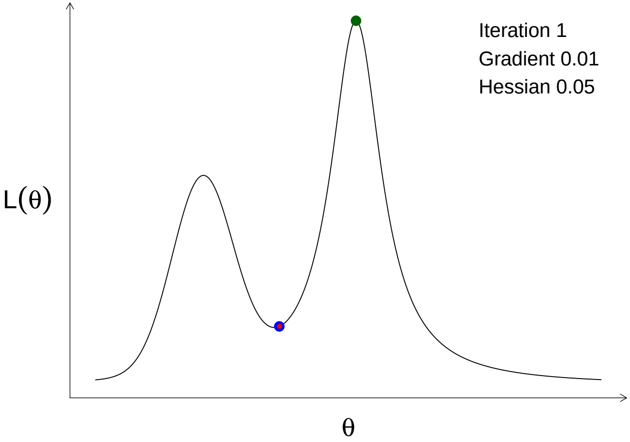

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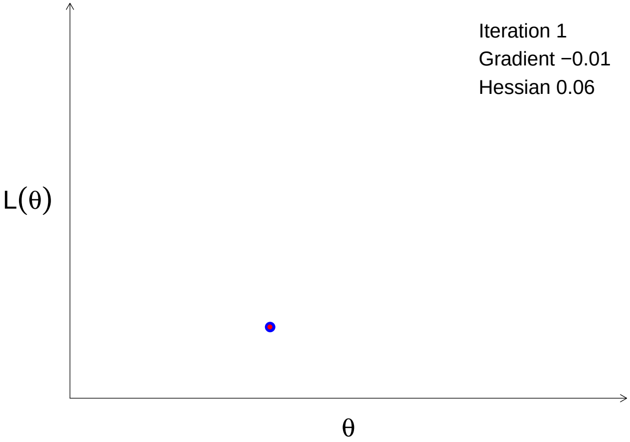

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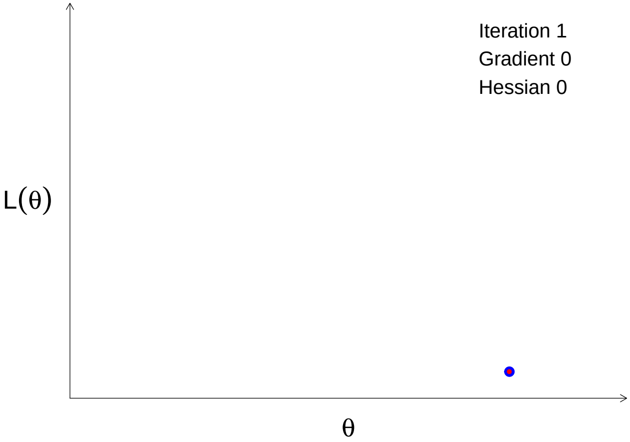
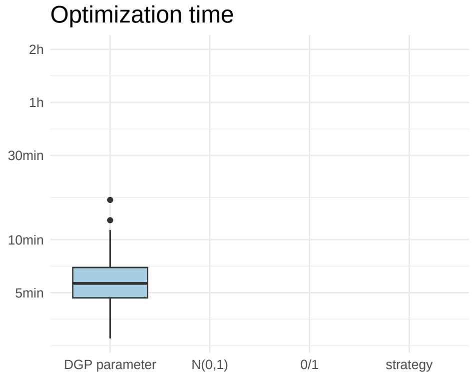


Outline of this talk

100 data sets simulated from a probit model with $\theta_i \sim N(0,1)$:

$N = 200$ deciders, $T = 10$ choices each, $J = 5$ alternatives, 5 covariates with normal random effects



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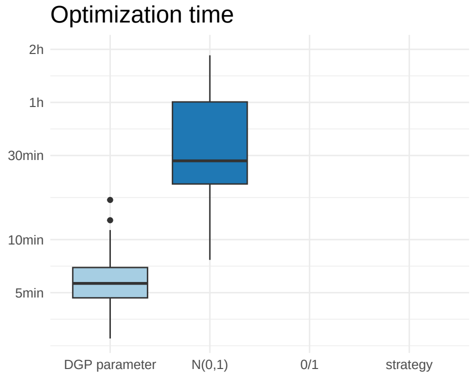

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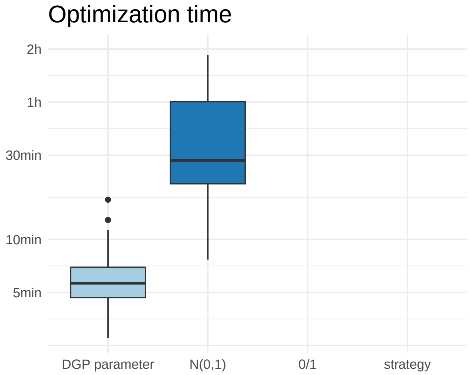

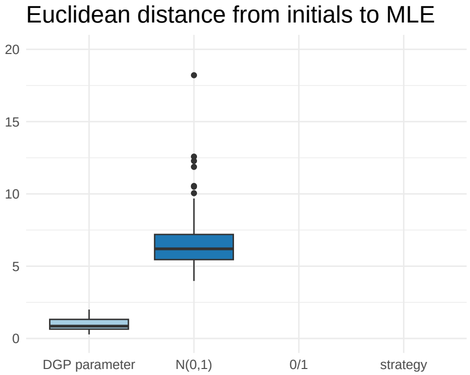

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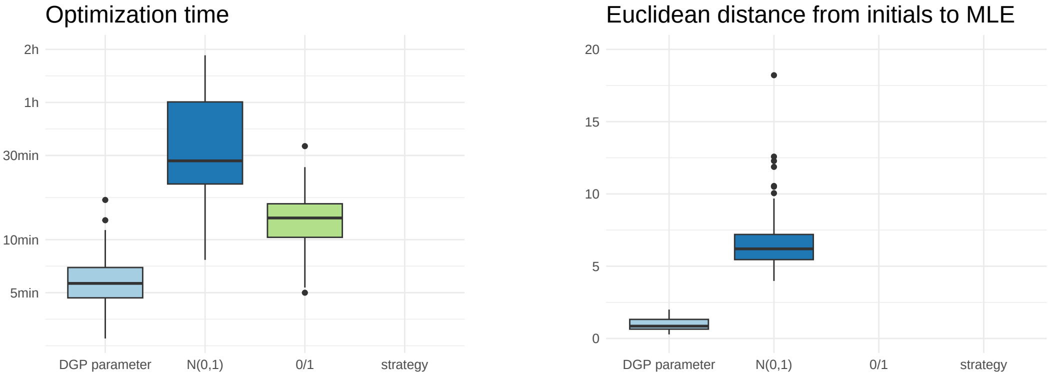

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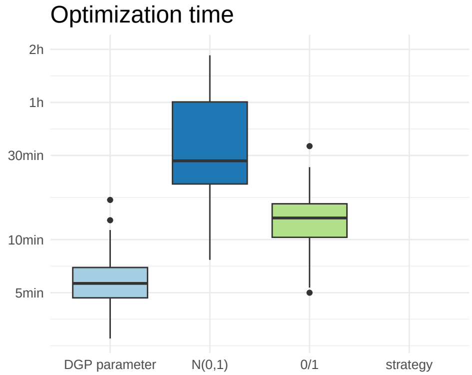

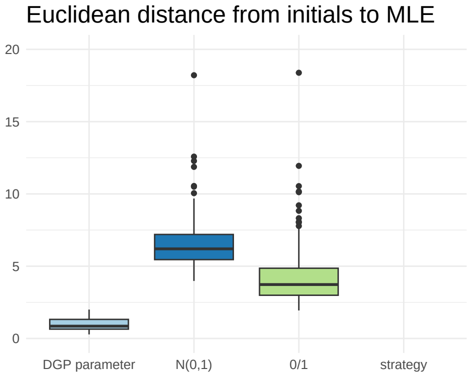

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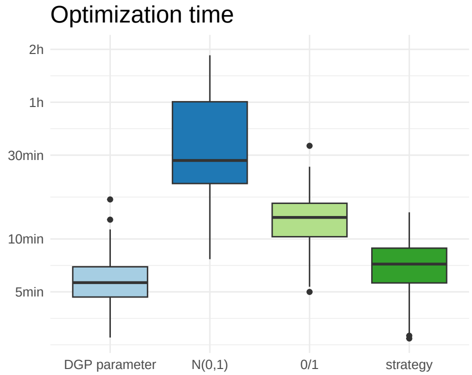

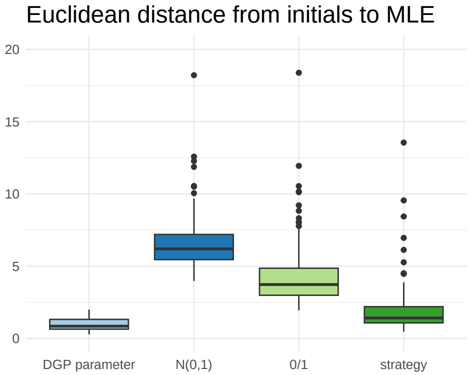

Outline

Different parameters, different strategies

$$\boldsymbol{U} = \boldsymbol{\mu} + \boldsymbol{X}(\boldsymbol{\beta} + \boldsymbol{\gamma}) + \boldsymbol{\varepsilon}$$

Parameter	Initialization strategy
$\boldsymbol{\beta}$ is alternative-varying and	
$\boldsymbol{X}$ is alternative-varying	constant utility direction
$\boldsymbol{X}$ is alternative-constant (ASCs $\boldsymbol{\mu}$ is special case)	minimize choice frequency prediction error
$\boldsymbol{\beta}$ is alternative-constant	linear probability model
covariance $\boldsymbol{\Sigma}$ for $\boldsymbol{\varepsilon}$	MCMC
covariance $\boldsymbol{\Omega}$ for $\boldsymbol{\gamma}$	MCMC

Different parameters, different strategies

$$\boldsymbol{U} = \mu + \boldsymbol{X}(\beta + \gamma) + \varepsilon$$

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▶ $\boldsymbol{X}$ is alternative-varying	constant utility direction
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β is alternative-constant	linear probability model
covariance Σ for ε	MCMC
covariance Ω for γ	MCMC

$$\boldsymbol{U} = \boldsymbol{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon}$$

$$\begin{pmatrix} U_1 \\ U_2 \end{pmatrix} = \begin{pmatrix} X_1 & 0 \\ 0 & X_2 \end{pmatrix} \begin{pmatrix} \beta_1 \\ \beta_2 \end{pmatrix} + \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \end{pmatrix}$$

$$y = \begin{cases} 1, & \text{if } \Delta U = U_1 - U_2 \geq 0 \\ 2, & \text{otherwise} \end{cases}$$

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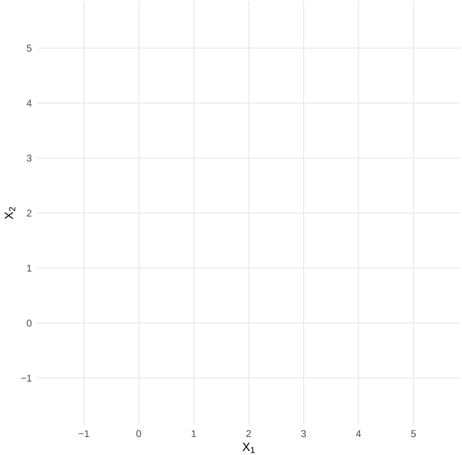

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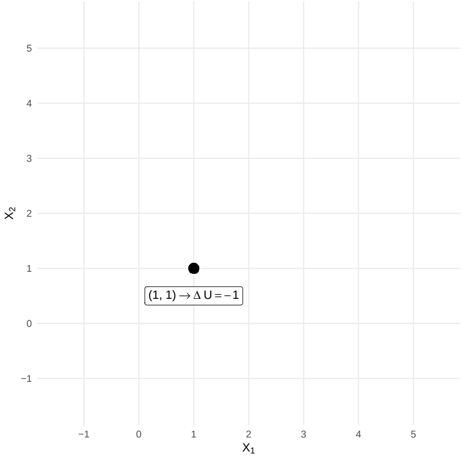

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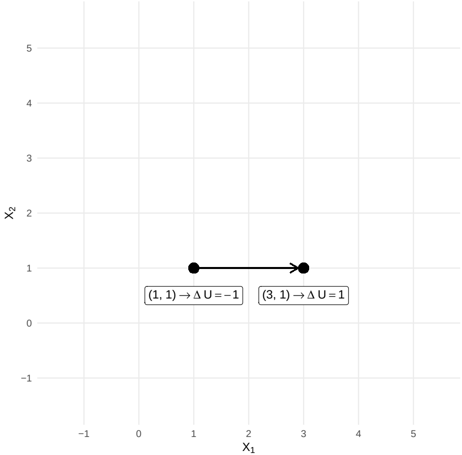

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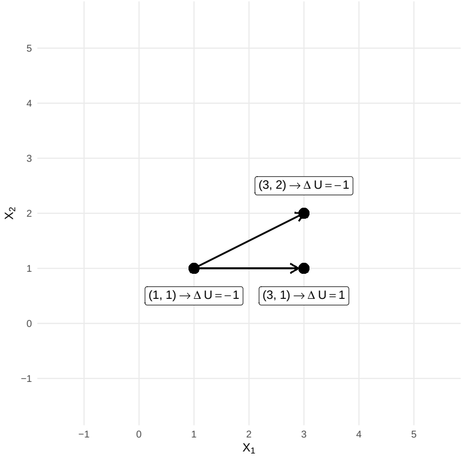

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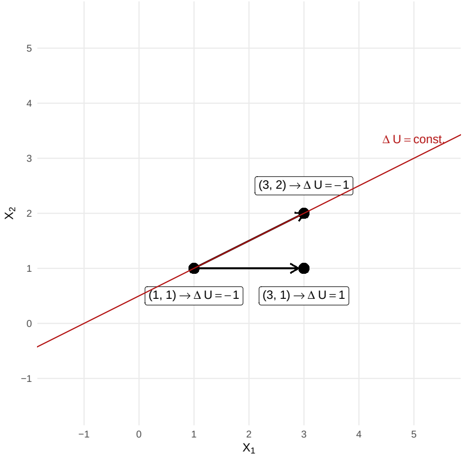

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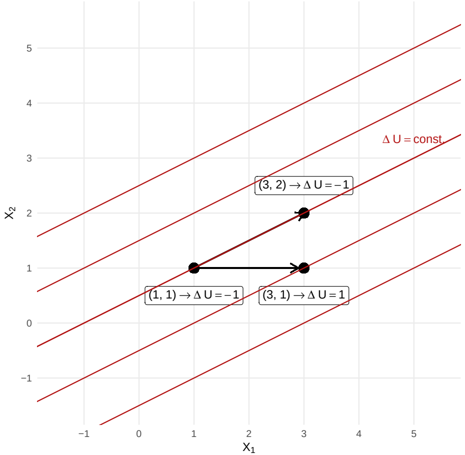

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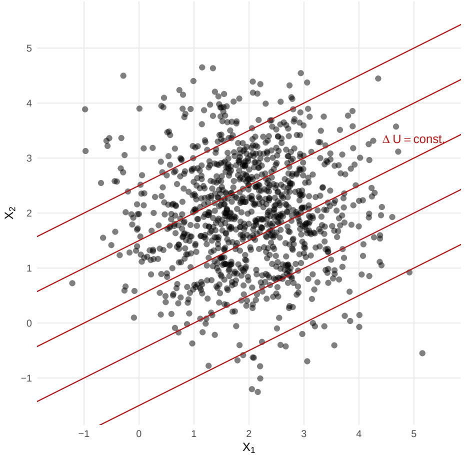

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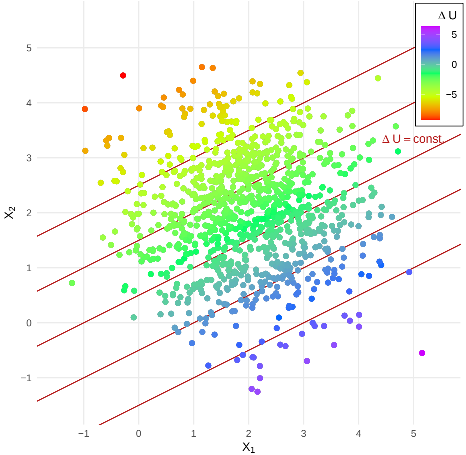

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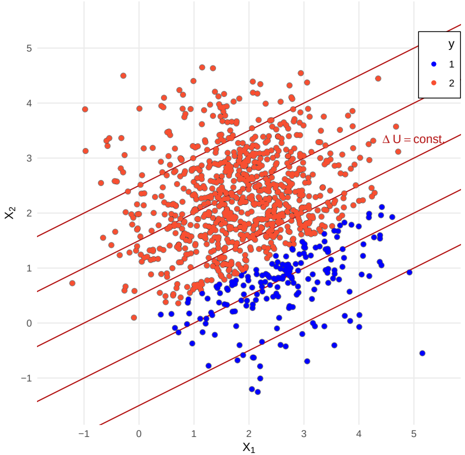

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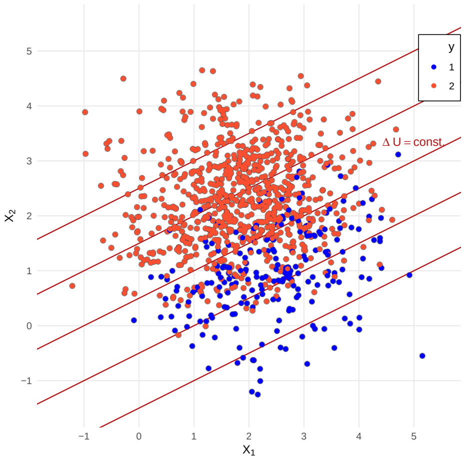

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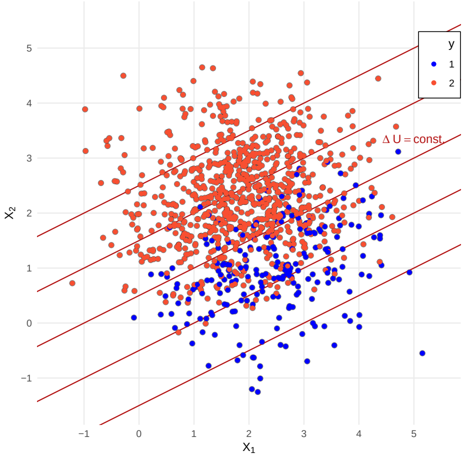
identify direction as the kernel of $\text{Cov}(y, \boldsymbol{X})$



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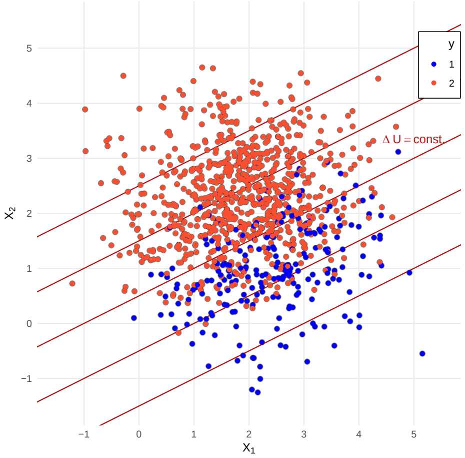
this yields a consistent initial estimator $\widehat{\beta}$



💡 Consistency as $N \to \infty$ and under a technical assumption on $\boldsymbol{X}$ (normality is sufficient)

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Simulation: $\beta_i \sim \mathcal{N}(0,1)$, 1.000 replications

N	1.000	■	10.000	□
J	4	■	8	□
$\boldsymbol{\Sigma}$	$\begin{pmatrix} 1 & 0 & \cdots \\ 0 & 1 & \\ \vdots & & \ddots \end{pmatrix}$	■	$\begin{pmatrix} 3 & 1 & \cdots \\ 1 & 3 & \\ \vdots & & \ddots \end{pmatrix}$	□

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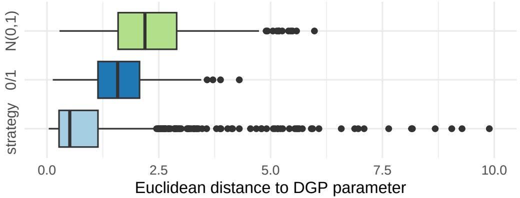
Average computation time: < 1 second



Consistency as $N \to \infty$ and under a technical assumption on $\boldsymbol{X}$ (normality is sufficient)

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$$\boldsymbol{U} = \boldsymbol{X}\beta + \varepsilon$$

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$$y = \begin{cases} 1, & \text{if } \Delta U = U_1 - U_2 \geq 0 \\ 2, & \text{otherwise} \end{cases}$$

$$\Delta U = \text{const. in direction } \overrightarrow{\begin{pmatrix} 1 \\ 0.5 \end{pmatrix}} = \overrightarrow{\begin{pmatrix} 1/\beta_1 \\ 1/\beta_2 \end{pmatrix}}$$

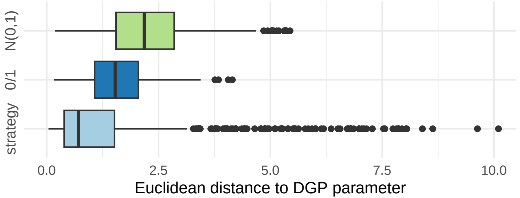
identify direction as the kernel of $\text{Cov}(y, \boldsymbol{X})$

this yields a consistent initial estimator $\widehat{\beta}$

Simulation: $\beta_i \sim \mathcal{N}(0,1)$, 1.000 replications

N	1.000	■	10.000	□
J	4	■	8	□
$\boldsymbol{\Sigma}$	$\begin{pmatrix} 1 & 0 & \cdots \\ 0 & 1 & \\ \vdots & & \ddots \end{pmatrix}$	□	$\begin{pmatrix} 3 & 1 & \cdots \\ 1 & 3 & \\ \vdots & & \ddots \end{pmatrix}$	■

Average computation time: < 1 second



💡 Consistency as $N \to \infty$ and under a technical assumption on $\boldsymbol{X}$ (normality is sufficient)

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$$\boldsymbol{U} = \boldsymbol{X}\beta + \varepsilon$$

$$\begin{pmatrix} U_1 \\ U_2 \end{pmatrix} = \begin{pmatrix} X_1 & 0 \\ 0 & X_2 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \end{pmatrix} + \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \end{pmatrix}$$

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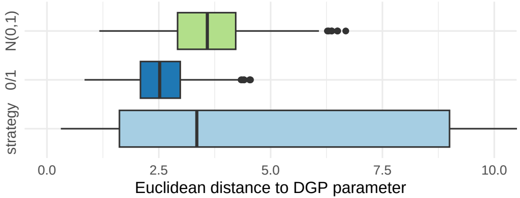
Simulation: $\beta_i \sim \mathcal{N}(0, 1)$, 1.000 replications

N	1.000	■	10.000	□
J	4	□	8	■
$\boldsymbol{\Sigma}$	$\begin{pmatrix} 1 & 0 & \cdots \\ 0 & 1 & \\ \vdots & & \ddots \end{pmatrix}$	■	$\begin{pmatrix} 3 & 1 & \cdots \\ 1 & 3 & \\ \vdots & & \ddots \end{pmatrix}$	□

Average computation time: <1 second



💡 Consistency as $N \to \infty$ and under a technical assumption on $\boldsymbol{X}$ (normality is sufficient)

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Simulation: $\beta_i \sim \mathcal{N}(0,1)$, 1.000 replications

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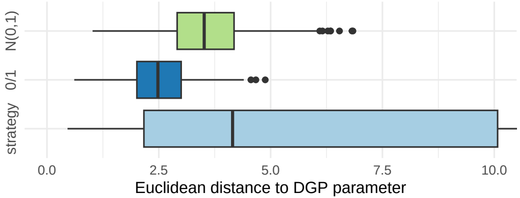
N	1.000	■	10.000	□
J	4	□	8	■
$\boldsymbol{\Sigma}$	$\begin{pmatrix} 1 & 0 & \cdots \\ 0 & 1 & \\ \vdots & & \ddots \end{pmatrix}$	□	$\begin{pmatrix} 3 & 1 & \cdots \\ 1 & 3 & \\ \vdots & & \ddots \end{pmatrix}$	■

Average computation time: < 1 second



Consistency as $N \to \infty$ and under a technical assumption on $\boldsymbol{X}$ (normality is sufficient)

$$\boldsymbol{U} = \boldsymbol{X}\beta + \varepsilon$$

$$\begin{pmatrix} U_1 \\ U_2 \end{pmatrix} = \begin{pmatrix} X_1 & 0 \\ 0 & X_2 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \end{pmatrix} + \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \end{pmatrix}$$

$$y = \begin{cases} 1, & \text{if } \Delta U = U_1 - U_2 \geq 0 \\ 2, & \text{otherwise} \end{cases}$$

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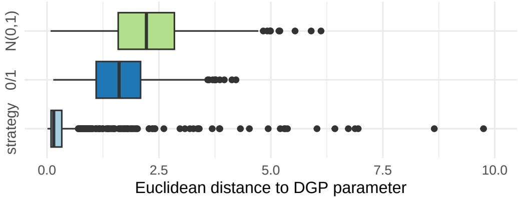
💡 Consistency as $N \to \infty$ and under a technical assumption on $\boldsymbol{X}$ (normality is sufficient)

Simulation: $\beta_i \sim \mathcal{N}(0,1)$, 1.000 replications

N	1.000	☐	10.000	■
J	4	■	8	☐
$\boldsymbol{\Sigma}$	$\begin{pmatrix} 1 & 0 & \cdots \\ 0 & 1 & \\ \vdots & & \ddots \end{pmatrix}$	■	$\begin{pmatrix} 3 & 1 & \cdots \\ 1 & 3 & \\ \vdots & & \ddots \end{pmatrix}$	☐

Average computation time: <1 second



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$$\boldsymbol{U} = \boldsymbol{X}\beta + \varepsilon$$

$$\begin{pmatrix} U_1 \\ U_2 \end{pmatrix} = \begin{pmatrix} X_1 & 0 \\ 0 & X_2 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \end{pmatrix} + \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \end{pmatrix}$$

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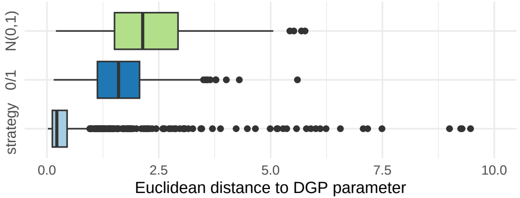
Simulation: $\beta_i \sim \mathcal{N}(0,1)$, 1.000 replications

N	1.000	☐	10.000	■
J	4	■	8	☐
$\boldsymbol{\Sigma}$	$\begin{pmatrix} 1 & 0 & \cdots \\ 0 & 1 & \\ \vdots & & \ddots \end{pmatrix}$	☐	$\begin{pmatrix} 3 & 1 & \cdots \\ 1 & 3 & \\ \vdots & & \ddots \end{pmatrix}$	■

Average computation time: < 1 second



Consistency as $N \to \infty$ and under a technical assumption on $\boldsymbol{X}$ (normality is sufficient)

$$\boldsymbol{U} = \boldsymbol{X}\beta + \varepsilon$$

$$\begin{pmatrix} U_1 \\ U_2 \end{pmatrix} = \begin{pmatrix} X_1 & 0 \\ 0 & X_2 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \end{pmatrix} + \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \end{pmatrix}$$

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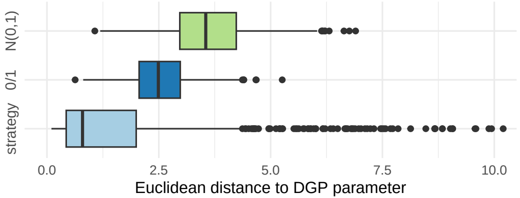
Simulation: $\beta_i \sim \mathcal{N}(0,1)$, 1.000 replications

N	1.000	☐	10.000	■
J	4	☐	8	■
$\boldsymbol{\Sigma}$	$\begin{pmatrix} 1 & 0 & \cdots \\ 0 & 1 & \\ \vdots & & \ddots \end{pmatrix}$	■	$\begin{pmatrix} 3 & 1 & \cdots \\ 1 & 3 & \\ \vdots & & \ddots \end{pmatrix}$	☐

Average computation time: $<$ 1 second



💡 Consistency as $N \to \infty$ and under a technical assumption on $\boldsymbol{X}$ (normality is sufficient)

$$\boldsymbol{U} = \boldsymbol{X}\beta + \varepsilon$$

$$\begin{pmatrix} U_1 \\ U_2 \end{pmatrix} = \begin{pmatrix} X_1 & 0 \\ 0 & X_2 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \end{pmatrix} + \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \end{pmatrix}$$

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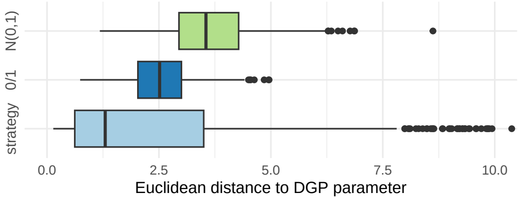
Simulation: $\beta_i \sim \mathcal{N}(0,1)$, 1.000 replications

N	1.000	□	10.000	■
J	4	□	8	■
$\boldsymbol{\Sigma}$	$\begin{pmatrix} 1 & 0 & \cdots \\ 0 & 1 & \\ \vdots & & \ddots \end{pmatrix}$	□	$\begin{pmatrix} 3 & 1 & \cdots \\ 1 & 3 & \\ \vdots & & \ddots \end{pmatrix}$	■

Average computation time: <1 second



Consistency as $N \to \infty$ and under a technical assumption on $\boldsymbol{X}$ (normality is sufficient)

$$\boldsymbol{U} = \mu + \boldsymbol{X}(\beta + \gamma) + \varepsilon$$

Parameter	Initialization strategy
β is alternative-varying and	
$\boldsymbol{X}$ is alternative-varying	constant utility direction
▶ $\boldsymbol{X}$ is alternative-constant (ASCs μ is special case)	minimize choice frequency prediction error
β is alternative-constant	linear probability model
covariance Σ for ε	MCMC
covariance Ω for γ	MCMC

$$\boldsymbol{U} = \boldsymbol{X}\beta + \varepsilon, \quad \boldsymbol{\varepsilon} \sim \mathcal{N}(0, \boldsymbol{\Sigma})$$

$$\begin{pmatrix} U_1 \\ U_2 \\ U_3 \end{pmatrix} = \begin{pmatrix} X & 0 & 0 \\ 0 & X & 0 \\ 0 & 0 & X \end{pmatrix} \begin{pmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \end{pmatrix} + \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \end{pmatrix}$$

$$\boldsymbol{U} = \boldsymbol{X}\beta + \varepsilon, \quad \varepsilon \sim \mathcal{N}(0, \boldsymbol{\Sigma})$$

$$\begin{pmatrix} U_1 \\ U_2 \\ U_3 \end{pmatrix} = \begin{pmatrix} X & 0 & 0 \\ 0 & X & 0 \\ 0 & 0 & X \end{pmatrix} \begin{pmatrix} 0 \\ \beta_2 \\ \beta_3 \end{pmatrix} + \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \end{pmatrix}$$

$$\boldsymbol{U} = \boldsymbol{X}\beta + \boldsymbol{\varepsilon}, \quad \boldsymbol{\varepsilon} \sim \mathcal{N}(0, \boldsymbol{\Sigma})$$

$$\begin{pmatrix} U_1 \\ U_2 \\ U_3 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ \beta_2 \\ \beta_3 \end{pmatrix} + \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \end{pmatrix}$$

$$\boldsymbol{U} = \boldsymbol{X}\beta + \varepsilon, \quad \varepsilon \sim \mathcal{N}(0, \boldsymbol{\Sigma})$$

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$$y = \begin{cases} 1, & \text{if } U_1 = \max \boldsymbol{U} \\ 2, & \text{if } U_2 = \max \boldsymbol{U} \\ 3, & \text{if } U_3 = \max \boldsymbol{U} \end{cases}$$

$$\boldsymbol{U} = \boldsymbol{X}\beta + \varepsilon, \quad \boldsymbol{\varepsilon} \sim \mathcal{N}(0, \boldsymbol{\Sigma})$$

$$\begin{pmatrix} U_1 \\ U_2 \\ U_3 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ \beta_2 \\ \beta_3 \end{pmatrix} + \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \end{pmatrix}$$

$$\boldsymbol{y}^d = \begin{cases} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, & \text{if } U_1 = \max \boldsymbol{U} \\ \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, & \text{if } U_2 = \max \boldsymbol{U} \\ \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, & \text{if } U_3 = \max \boldsymbol{U} \end{cases}$$

$$\boldsymbol{U} = \boldsymbol{X}\beta + \varepsilon, \quad \boldsymbol{\varepsilon} \sim \mathcal{N}(0, \boldsymbol{\Sigma})$$

$$\begin{pmatrix} U_1 \\ U_2 \\ U_3 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ \beta_2 \\ \beta_3 \end{pmatrix} + \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \end{pmatrix}$$

Initialization strategy:

1. assume $\boldsymbol{\Sigma}$ is known, or choose $\boldsymbol{\Sigma} = I_J$ else
2. let $\overline{\boldsymbol{y}^d}$ be the average of $\boldsymbol{y}^d$
3. find $\widehat{\boldsymbol{\beta}} = \arg\min_{\boldsymbol{\beta}} \| P(\boldsymbol{\beta}, \boldsymbol{\Sigma}) - \overline{\boldsymbol{y}^d} \|_2$

$$\boldsymbol{U} = \boldsymbol{X}\beta + \varepsilon, \quad \varepsilon \sim \mathcal{N}(0, \boldsymbol{\Sigma})$$

$$\begin{pmatrix} U_1 \\ U_2 \\ U_3 \end{pmatrix} = \begin{pmatrix} X & 0 & 0 \\ 0 & X & 0 \\ 0 & 0 & X \end{pmatrix} \begin{pmatrix} 0 \\ \beta_2 \\ \beta_3 \end{pmatrix} + \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \end{pmatrix}$$

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Initialization strategy:

1. assume Σ is known, or choose $\Sigma = I_J$ else
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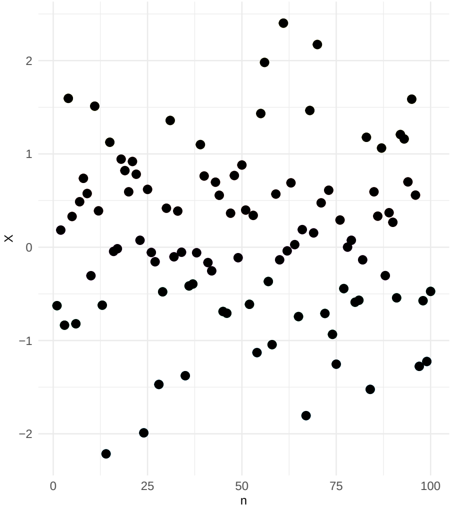
3. find $\widehat{\beta} = \arg\min_{\beta} \|P(\beta, \Sigma) - \overline{\boldsymbol{y}^d}\|_2$



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$$\begin{pmatrix} U_1 \\ U_2 \\ U_3 \end{pmatrix} = \begin{pmatrix} X & 0 & 0 \\ 0 & X & 0 \\ 0 & 0 & X \end{pmatrix} \begin{pmatrix} 0 \\ \beta_2 \\ \beta_3 \end{pmatrix} + \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \end{pmatrix}$$

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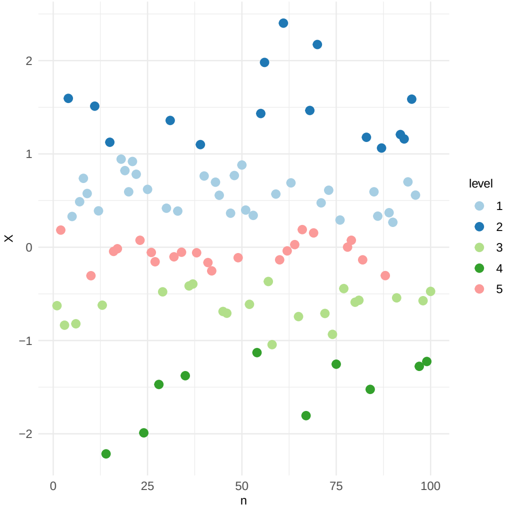

$$\boldsymbol{U} = \boldsymbol{X}\beta + \varepsilon, \quad \varepsilon \sim \mathcal{N}(0, \boldsymbol{\Sigma})$$

$$\begin{pmatrix} U_1 \\ U_2 \\ U_3 \end{pmatrix} = \begin{pmatrix} X & 0 & 0 \\ 0 & X & 0 \\ 0 & 0 & X \end{pmatrix} \begin{pmatrix} 0 \\ \beta_2 \\ \beta_3 \end{pmatrix} + \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \end{pmatrix}$$

Initialization strategy:

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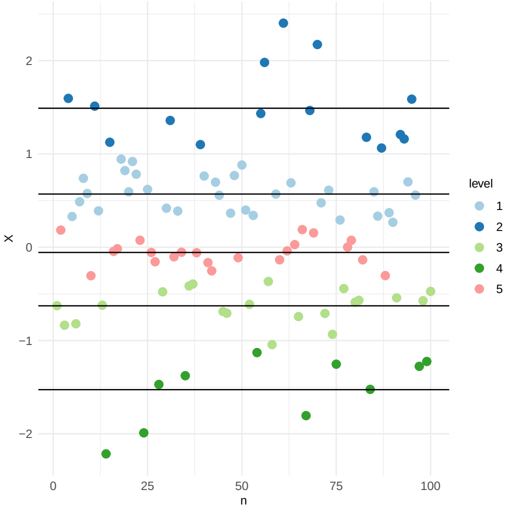
3. find $\widehat{\beta} = \arg\min_{\beta} \| P(\beta, \boldsymbol{\Sigma}) - \overline{\boldsymbol{y}^d} \|_2$



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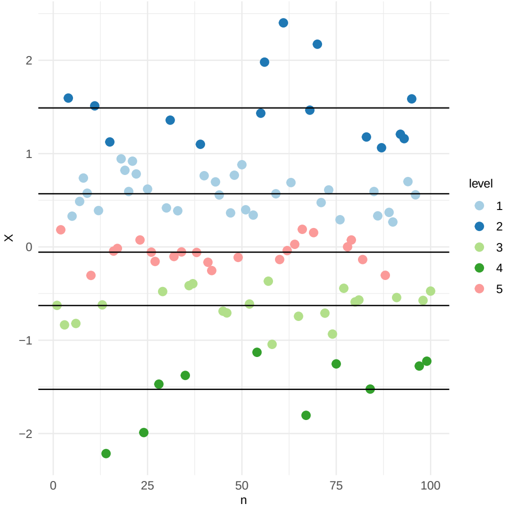
Initialization strategy:

1. assume $\boldsymbol{\Sigma}$ is known, or choose $\boldsymbol{\Sigma} = I_J$ else
2. localize X, for each level $L_i \neq 0$:
   - 2.1 let $\overline{\boldsymbol{y}_i^d}$ be the average of $\boldsymbol{y}_i^d$
   - 2.2 find $\widehat{\boldsymbol{\beta}}_i = \arg\min_{\boldsymbol{\beta}} \|P(\boldsymbol{\beta}, \boldsymbol{\Sigma}) - \overline{\boldsymbol{y}_i^d}\|_2$
   - 2.3 $\widehat{\boldsymbol{\beta}}_i \leftarrow \widehat{\boldsymbol{\beta}}_i / L_i$
3. $\widehat{\boldsymbol{\beta}} = \overline{\widehat{\boldsymbol{\beta}}_i}$



$$\boldsymbol{U} = \boldsymbol{X}\beta + \varepsilon, \quad \varepsilon \sim \mathcal{N}(0, \Sigma)$$

$$\begin{pmatrix} U_1 \\ U_2 \\ U_3 \end{pmatrix} = \begin{pmatrix} X & 0 & 0 \\ 0 & X & 0 \\ 0 & 0 & X \end{pmatrix} \begin{pmatrix} 0 \\ \beta_2 \\ \beta_3 \end{pmatrix} + \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \end{pmatrix}$$

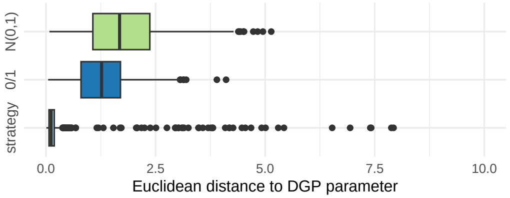
Initialization strategy:

1. assume Σ is known, or choose $\Sigma = I_J$ else
2. localize X, for each level $L_i \neq 0$:
   2.1 let $\overline{\boldsymbol{y}_i^d}$ be the average of $\boldsymbol{y}_i^d$
   2.2 find $\widehat{\beta}_i = \arg\min_{\beta} \|P(\beta, \Sigma) - \overline{\boldsymbol{y}_i^d}\|_2$
   2.3 $\widehat{\beta}_i \leftarrow \widehat{\beta}_i / L_i$
3. $\widehat{\beta} = \overline{\widehat{\beta}_i}$

Simulation: $\beta_i \sim \mathcal{N}(0,1)$, $N = 1.000$, 1.000 rep.

J	4	■	8	□
levels	1	■	N	□
$\boldsymbol{\Sigma}$	$\begin{pmatrix} 1 & 0 & \cdots \\ 0 & 1 & \\ \vdots & & \ddots \end{pmatrix}$	■	$\begin{pmatrix} 3 & 1 & \cdots \\ 1 & 3 & \\ \vdots & & \ddots \end{pmatrix}$	□

Average computation time: < 1 second



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$$\begin{pmatrix} U_1 \\ U_2 \\ U_3 \end{pmatrix} = \begin{pmatrix} X & 0 & 0 \\ 0 & X & 0 \\ 0 & 0 & X \end{pmatrix} \begin{pmatrix} 0 \\ \beta_2 \\ \beta_3 \end{pmatrix} + \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \end{pmatrix}$$

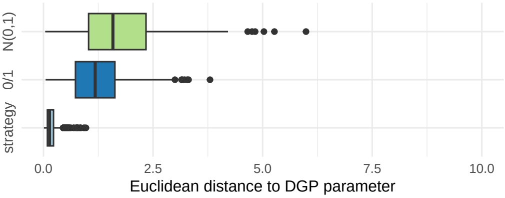
Initialization strategy:

1. assume Σ is known, or choose $\Sigma = I_J$ else
2. localize X, for each level $L_i \neq 0$:
   - 2.1 let $\overline{\boldsymbol{y}_i^d}$ be the average of $\boldsymbol{y}_i^d$
   - 2.2 find $\widehat{\beta}_i = \arg\min_{\beta} \|P(\beta, \Sigma) - \overline{\boldsymbol{y}_i^d}\|_2$
   - 2.3 $\widehat{\beta}_i \leftarrow \widehat{\beta}_i / L_i$
3. $\widehat{\beta} = \overline{\widehat{\beta}_i}$

Simulation: $\beta_i \sim \mathcal{N}(0,1)$, $N = 1.000$, 1.000 rep.

J	4	■	8	□
levels	1	■	N	□
$\boldsymbol{\Sigma}$	$\begin{pmatrix} 1 & 0 & \cdots \\ 0 & 1 & \\ \vdots & & \ddots \end{pmatrix}$	□	$\begin{pmatrix} 3 & 1 & \cdots \\ 1 & 3 & \\ \vdots & & \ddots \end{pmatrix}$	■

Average computation time: < 1 second



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$$U = X\beta + \varepsilon, \quad \varepsilon \sim \mathcal{N}(0, \Sigma)$$

$$\begin{pmatrix} U_1 \\ U_2 \\ U_3 \end{pmatrix} = \begin{pmatrix} X & 0 & 0 \\ 0 & X & 0 \\ 0 & 0 & X \end{pmatrix} \begin{pmatrix} 0 \\ \beta_2 \\ \beta_3 \end{pmatrix} + \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \end{pmatrix}$$

Initialization strategy:

1. assume Σ is known, or choose $\Sigma = I_J$ else
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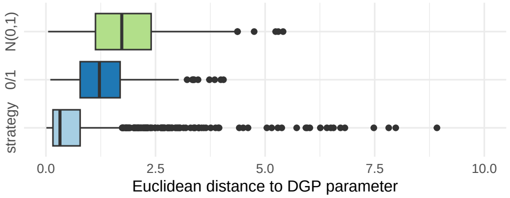
   - 2.1 let $\overline{y_i^d}$ be the average of y_i^d
   - 2.2 find $\widehat{\beta}_i = \arg\min_{\beta} \|P(\beta, \Sigma) - \overline{y_i^d}\|_2$
   - 2.3 $\widehat{\beta}_i \leftarrow \widehat{\beta}_i / L_i$
3. $\widehat{\beta} = \overline{\widehat{\beta}_i}$

Simulation: $\beta_i \sim \mathcal{N}(0, 1)$, $N = 1.000$, 1.000 rep.

J	4	■	8	□
levels	1	□	N	■
Σ	$\begin{pmatrix} 1 & 0 & \cdots \\ 0 & 1 & \\ \vdots & & \ddots \end{pmatrix}$	■	$\begin{pmatrix} 3 & 1 & \cdots \\ 1 & 3 & \\ \vdots & & \ddots \end{pmatrix}$	□

Average computation time: < 1 second



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$$U = X\beta + \varepsilon, \quad \varepsilon \sim \mathcal{N}(0, \Sigma)$$

$$\begin{pmatrix} U_1 \\ U_2 \\ U_3 \end{pmatrix} = \begin{pmatrix} X & 0 & 0 \\ 0 & X & 0 \\ 0 & 0 & X \end{pmatrix} \begin{pmatrix} 0 \\ \beta_2 \\ \beta_3 \end{pmatrix} + \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \end{pmatrix}$$

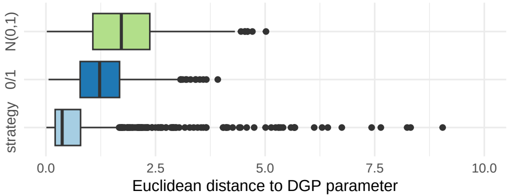
Initialization strategy:

1. assume Σ is known, or choose $\Sigma = I_J$ else
2. localize X, for each level $L_i \neq 0$:
   2.1 let $\overline{y_i^d}$ be the average of y_i^d
   2.2 find $\widehat{\beta}_i = \arg\min_{\beta} \|P(\beta, \Sigma) - \overline{y_i^d}\|_2$
   2.3 $\widehat{\beta}_i \leftarrow \widehat{\beta}_i / L_i$
3. $\widehat{\beta} = \overline{\widehat{\beta}_i}$

Simulation: $\beta_i \sim \mathcal{N}(0,1)$, $N = 1.000$, 1.000 rep.

J	4	■	8	□
levels	1	□	N	■
Σ	$\begin{pmatrix} 1 & 0 & \cdots \\ 0 & 1 & \\ \vdots & & \ddots \end{pmatrix}$	□	$\begin{pmatrix} 3 & 1 & \cdots \\ 1 & 3 & \\ \vdots & & \ddots \end{pmatrix}$	■

Average computation time: < 1 second



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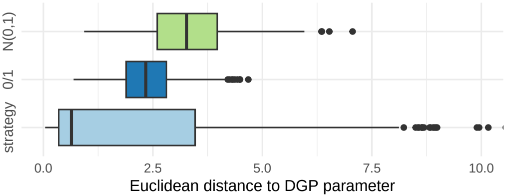
Simulation: $\beta_i \sim \mathcal{N}(0,1)$, $N = 1.000$, 1.000 rep.

J	4	☐	8	■
levels	1	■	N	☐
Σ	$\begin{pmatrix} 1 & 0 & \cdots \\ 0 & 1 & \\ \vdots & & \ddots \end{pmatrix}$	■	$\begin{pmatrix} 3 & 1 & \cdots \\ 1 & 3 & \\ \vdots & & \ddots \end{pmatrix}$	☐

Average computation time: 2 seconds



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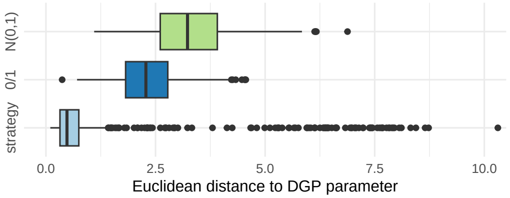
J	4	☐	8	■
levels	1	■	N	☐
Σ	$\begin{pmatrix} 1 & 0 & \cdots \\ 0 & 1 & \\ \vdots & & \ddots \end{pmatrix}$	☐	$\begin{pmatrix} 3 & 1 & \cdots \\ 1 & 3 & \\ \vdots & & \ddots \end{pmatrix}$	■

Average computation time: 2 seconds



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Simulation: $\beta_i \sim \mathcal{N}(0, 1)$, $N = 1.000$, 1.000 rep.

J	4	☐	8	■
levels	1	☐	N	■
Σ	$\begin{pmatrix} 1 & 0 & \cdots \\ 0 & 1 & \\ \vdots & & \ddots \end{pmatrix}$	■	$\begin{pmatrix} 3 & 1 & \cdots \\ 1 & 3 & \\ \vdots & & \ddots \end{pmatrix}$	☐

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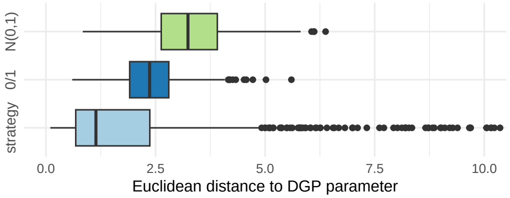

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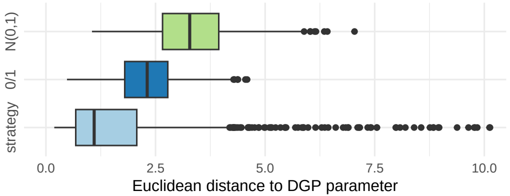
J	4	☐	8	■
levels	1	☐	N	■
Σ	$\begin{pmatrix} 1 & 0 & \cdots \\ 0 & 1 & \\ \vdots & & \ddots \end{pmatrix}$	☐	$\begin{pmatrix} 3 & 1 & \cdots \\ 1 & 3 & \\ \vdots & & \ddots \end{pmatrix}$	■

Average computation time: 2 seconds



$$\boldsymbol{U} = \boldsymbol{\mu} + \boldsymbol{X}(\boldsymbol{\beta} + \boldsymbol{\gamma}) + \boldsymbol{\varepsilon}$$

Parameter	Initialization strategy
$\boldsymbol{\beta}$ is alternative-varying and	
$\boldsymbol{X}$ is alternative-varying	constant utility direction
$\boldsymbol{X}$ is alternative-constant (ASCs $\boldsymbol{\mu}$ is special case)	minimize choice frequency prediction error
► $\boldsymbol{\beta}$ is alternative-constant	linear probability model
covariance Σ for $\boldsymbol{\varepsilon}$	MCMC
covariance Ω for $\boldsymbol{\gamma}$	MCMC

$$\boldsymbol{U} = \boldsymbol{X}\beta + \varepsilon, \quad \varepsilon \sim \mathcal{N}(0, \boldsymbol{\Sigma})$$

$$\boldsymbol{U} \;=\; \boldsymbol{X}\boldsymbol{\beta} \;+\; \boldsymbol{\varepsilon}, \quad \boldsymbol{\varepsilon} \sim \mathcal{N}(0, \boldsymbol{\Sigma})$$

$$\underbrace{\begin{pmatrix} U_1 \\ U_2 \end{pmatrix}}_{J=2} = \begin{pmatrix} X_1 \\ X_2 \end{pmatrix} \underbrace{\beta_1}_{P=1} + \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \end{pmatrix}$$

$$\boldsymbol{U} = \boldsymbol{X}\beta + \varepsilon, \quad \varepsilon \sim \mathcal{N}(0, \boldsymbol{\Sigma})$$

$$\underbrace{\begin{pmatrix} U_1 \\ U_2 \end{pmatrix}}_{J=2} = \begin{pmatrix} X_{1,1} & X_{1,2} \\ X_{2,1} & X_{2,2} \end{pmatrix} \underbrace{\begin{pmatrix} \beta_1 \\ \beta_2 \end{pmatrix}}_{P=2} + \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \end{pmatrix}$$

$$\boldsymbol{U} = \boldsymbol{X}\beta + \varepsilon, \quad \boldsymbol{\varepsilon} \sim \mathcal{N}(0, \boldsymbol{\Sigma})$$

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$$y = \begin{cases} 1, & \text{if } U_1 = \max \boldsymbol{U} \\ 2, & \text{if } U_2 = \max \boldsymbol{U} \end{cases}$$

$$\boldsymbol{U} = \boldsymbol{X}\beta + \varepsilon, \quad \boldsymbol{\varepsilon} \sim \mathcal{N}(0, \boldsymbol{\Sigma})$$

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$$\boldsymbol{y}^d = \begin{cases} \binom{1}{0}, & \text{if } U_1 = \max \boldsymbol{U} \\ \binom{0}{1}, & \text{if } U_2 = \max \boldsymbol{U} \end{cases}$$

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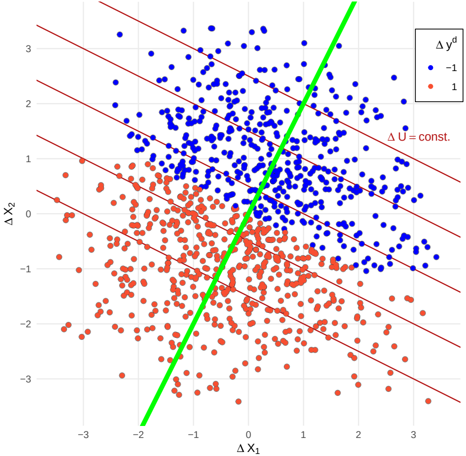
$$\boldsymbol{y}^d = \begin{cases} \begin{pmatrix} 1 \\ 0 \end{pmatrix}, & \text{if } U_1 = \max \boldsymbol{U} \\ \begin{pmatrix} 0 \\ 1 \end{pmatrix}, & \text{if } U_2 = \max \boldsymbol{U} \end{cases}$$



$$\boldsymbol{U} = \boldsymbol{X}\beta + \varepsilon, \quad \varepsilon \sim \mathcal{N}(0, \boldsymbol{\Sigma})$$

$$\boldsymbol{y}^d = \boldsymbol{X}\alpha + \boldsymbol{u}, \quad \boldsymbol{u} \sim \mathcal{N}(0, \boldsymbol{I})$$

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$$\boldsymbol{y}^d = \boldsymbol{X}\boldsymbol{\alpha} + \boldsymbol{u}, \quad \boldsymbol{u} \sim \mathcal{N}(0, \boldsymbol{I})$$

- find $\widehat{\boldsymbol{\alpha}}$ via OLS
- $\widehat{\boldsymbol{\alpha}}/\|\widehat{\boldsymbol{\alpha}}\|$ is consistent for $\boldsymbol{\beta}/\|\boldsymbol{\beta}\|$

  Consistency as $N \to \infty$ and under a technical assumption on $\boldsymbol{X}$ (normality is sufficient)
- robust to $\boldsymbol{\Sigma}$ and the existence of additional independent regressors

$$\boldsymbol{U} = \boldsymbol{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon}, \quad \boldsymbol{\varepsilon} \sim \mathcal{N}(0, \boldsymbol{\Sigma})$$

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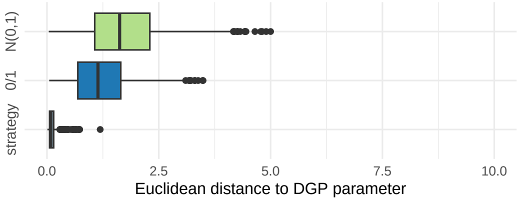
Simulation: $\beta_i \sim \mathcal{N}(0, 1)$, $J = 3$, 1.000 replications

N	1.000	■	10.000	□
P	3	■	10	□
$\boldsymbol{\Sigma}$	$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$	■	$\begin{pmatrix} 3 & 1 & 1 \\ 1 & 3 & 1 \\ 1 & 1 & 3 \end{pmatrix}$	□

Average computation time: <1 second



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$\epsilon \sim \Sigma$

$$\boldsymbol{U} = \boldsymbol{X}\beta + \varepsilon, \quad \varepsilon \sim \mathcal{N}(0, \Sigma)$$

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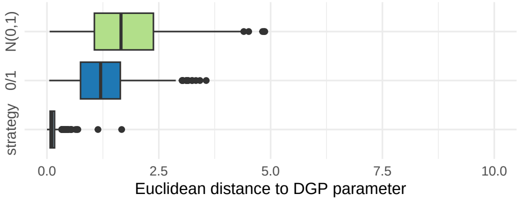
Simulation: $\beta_i \sim \mathcal{N}(0,1)$, $J = 3$, 1.000 replications

N	1.000	■	10.000	☐
P	3	■	10	☐
Σ	$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$	☐	$\begin{pmatrix} 3 & 1 & 1 \\ 1 & 3 & 1 \\ 1 & 1 & 3 \end{pmatrix}$	■

Average computation time: $<$ 1 second



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Simulation: $\beta_i \sim \mathcal{N}(0,1)$, $J = 3$, 1.000 replications

N	1.000	■	10.000	□
P	3	□	10	■
$\boldsymbol{\Sigma}$	$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$	■	$\begin{pmatrix} 3 & 1 & 1 \\ 1 & 3 & 1 \\ 1 & 1 & 3 \end{pmatrix}$	□

$$\boldsymbol{U} = \boldsymbol{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon}, \quad \boldsymbol{\varepsilon} \sim \mathcal{N}(0, \boldsymbol{\Sigma})$$

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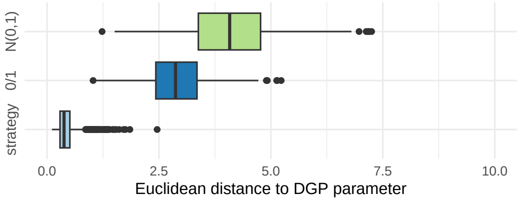
Average computation time: < 1 second



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N	1.000	■	10.000	☐
P	3	☐	10	■
Σ	$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$	☐	$\begin{pmatrix} 3 & 1 & 1 \\ 1 & 3 & 1 \\ 1 & 1 & 3 \end{pmatrix}$	■

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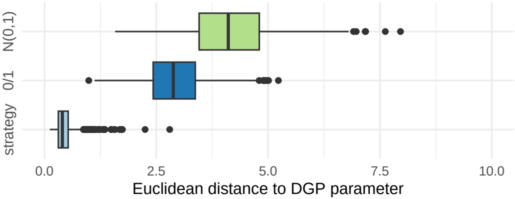

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N	1.000	☐	10.000	■
P	3	■	10	☐
Σ	$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$	■	$\begin{pmatrix} 3 & 1 & 1 \\ 1 & 3 & 1 \\ 1 & 1 & 3 \end{pmatrix}$	☐

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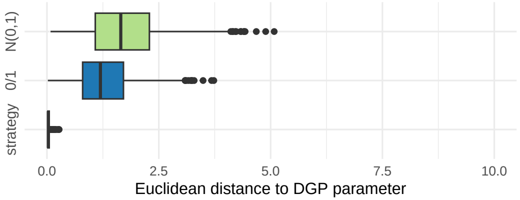

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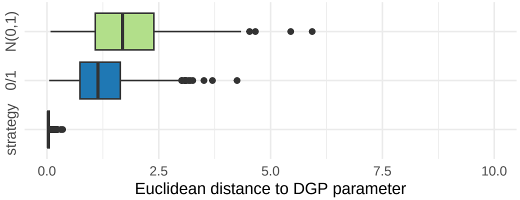
Simulation: $\beta_i \sim \mathcal{N}(0,1)$, $J = 3$, 1.000 replications

N	1.000	☐	10.000	■
P	3	■	10	☐
$\boldsymbol{\Sigma}$	$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$	☐	$\begin{pmatrix} 3 & 1 & 1 \\ 1 & 3 & 1 \\ 1 & 1 & 3 \end{pmatrix}$	■

Average computation time: <1 second



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N	1.000	☐	10.000	■
P	3	☐	10	■
$\boldsymbol{\Sigma}$	$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$	■	$\begin{pmatrix} 3 & 1 & 1 \\ 1 & 3 & 1 \\ 1 & 1 & 3 \end{pmatrix}$	☐

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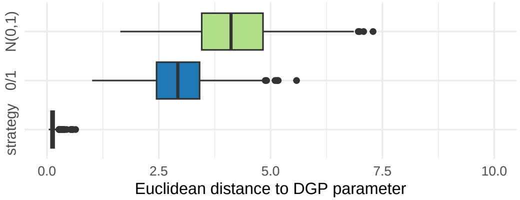

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Simulation: $\beta_i \sim \mathcal{N}(0,1)$, $J = 3$, 1.000 replications

N	1.000	☐	10.000	■
P	3	☐	10	■
$\boldsymbol{\Sigma}$	$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$	☐	$\begin{pmatrix} 3 & 1 & 1 \\ 1 & 3 & 1 \\ 1 & 1 & 3 \end{pmatrix}$	■

$$\boldsymbol{U} = \boldsymbol{X}\beta + \varepsilon, \quad \varepsilon \sim \mathcal{N}(0, \boldsymbol{\Sigma})$$

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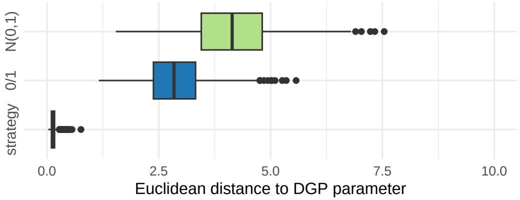
Average computation time: <1 second

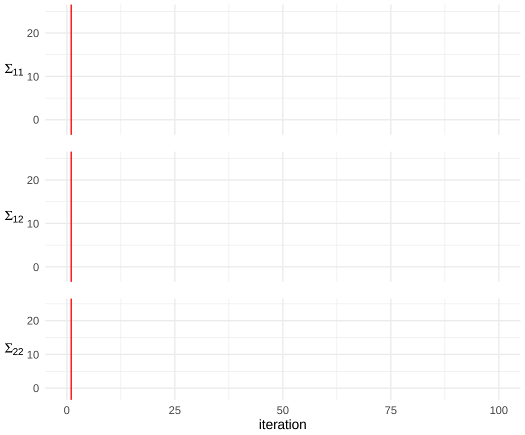


$$\boldsymbol{U} = \boldsymbol{\mu} + \boldsymbol{X}(\beta + \gamma) + \varepsilon$$

Parameter	Initialization strategy
β is alternative-varying and	
$\boldsymbol{X}$ is alternative-varying	constant utility direction
$\boldsymbol{X}$ is alternative-constant (ASCs $\boldsymbol{\mu}$ is special case)	minimize choice frequency prediction error
β is alternative-constant	linear probability model
▶ covariance Σ for ε	MCMC
covariance Ω for γ	MCMC

$$\boldsymbol{U} = \boldsymbol{X}\boldsymbol{\beta} + \varepsilon, \quad \varepsilon \sim \mathcal{N}(0, \boldsymbol{\Sigma})$$

1. assume $\boldsymbol{\beta}$ is known
2. for $\approx$ 100 iterations:
   2.1 $\boldsymbol{U} \mid \boldsymbol{\Sigma}, \boldsymbol{\beta}, \boldsymbol{X}, y \sim$ truncated $\mathcal{N}$
   2.2 $\boldsymbol{\Sigma} \mid \boldsymbol{\beta}, \boldsymbol{U} \sim \mathcal{W}^{-1}$
3. derive $\widehat{\boldsymbol{\Sigma}}$ as sample average of $\boldsymbol{\Sigma}$



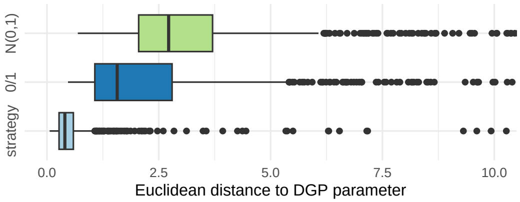
$$\boldsymbol{U} = \boldsymbol{X}\beta + \varepsilon, \quad \varepsilon \sim \mathcal{N}(0, \Sigma)$$

1. assume β is known
2. for $\approx$ 100 iterations:
   2.1 $\boldsymbol{U} \mid \Sigma, \beta, \boldsymbol{X}, y \sim$ truncated $\mathcal{N}$
   2.2 $\Sigma \mid \beta, \boldsymbol{U} \sim \mathcal{W}^{-1}$
3. derive $\widehat{\Sigma}$ as sample average of Σ

Simulation: $\boldsymbol{\Sigma} \sim \mathcal{W}^{-1}(J \cdot \boldsymbol{I}, J+1)$, $P = 5$, 1.000 rep.

N	1.000	■	10.000	□
J	4	■	8	□

Average computation time: 3 seconds



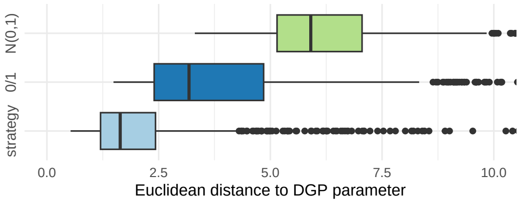
$$U = X\beta + \varepsilon, \quad \varepsilon \sim \mathcal{N}(0, \Sigma)$$

1. assume β is known
2. for ≈ 100 iterations:
   2.1 $U \mid \Sigma, \beta, X, y \sim$ truncated $\mathcal{N}$
   2.2 $\Sigma \mid \beta, U \sim \mathcal{W}^{-1}$
3. derive $\hat{\Sigma}$ as sample average of Σ

Simulation: $\Sigma \sim \mathcal{W}^{-1}(J \cdot I, J + 1)$, $P = 5$, 1.000 rep.

N	1.000	■	10.000	□
J	4	□	8	■

Average computation time: 3 seconds



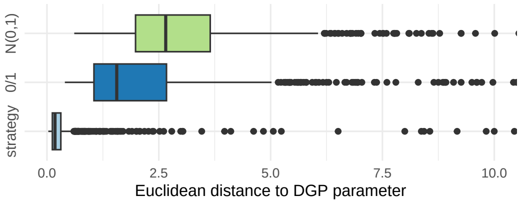
Simulation: $\boldsymbol{\Sigma} \sim \mathcal{W}^{-1}(J \cdot \boldsymbol{I}, J+1)$, $P = 5$, 1.000 rep.

N	1.000	☐	10.000	■
J	4	■	8	☐

$$\boldsymbol{U} = \boldsymbol{X}\beta + \varepsilon, \quad \varepsilon \sim \mathcal{N}(0, \Sigma)$$

Average computation time: 30 seconds

1. assume β is known
2. for ≈ 100 iterations:
   2.1 $\boldsymbol{U} \mid \Sigma, \beta, \boldsymbol{X}, y \sim$ truncated $\mathcal{N}$
   2.2 $\Sigma \mid \beta, \boldsymbol{U} \sim \mathcal{W}^{-1}$
3. derive $\widehat{\Sigma}$ as sample average of Σ



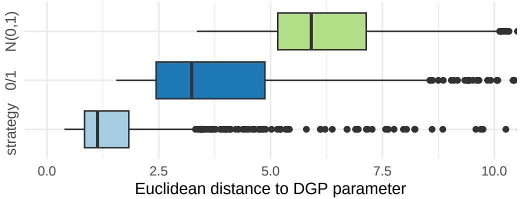
$$\boldsymbol{U} = \boldsymbol{X}\beta + \varepsilon, \quad \varepsilon \sim \mathcal{N}(0, \Sigma)$$

1. assume β is known
2. for $\approx$ 100 iterations:
   2.1 $\boldsymbol{U} \mid \Sigma, \beta, \boldsymbol{X}, y \sim$ truncated $\mathcal{N}$
   2.2 $\Sigma \mid \beta, \boldsymbol{U} \sim \mathcal{W}^{-1}$
3. derive $\widehat{\Sigma}$ as sample average of Σ

Simulation: $\boldsymbol{\Sigma} \sim \mathcal{W}^{-1}(J \cdot \boldsymbol{I}, J + 1)$, $P = 5$, 1.000 rep.

N	1.000	☐	10.000	■
J	4	☐	8	■

Average computation time: 30 seconds



$$\boldsymbol{U} = \boldsymbol{\mu} + \boldsymbol{X}(\boldsymbol{\beta} + \boldsymbol{\gamma}) + \boldsymbol{\varepsilon}$$

Parameter	Initialization strategy
$\boldsymbol{\beta}$ is alternative-varying and	
$\boldsymbol{X}$ is alternative-varying	constant utility direction
$\boldsymbol{X}$ is alternative-constant (ASCs $\boldsymbol{\mu}$ is special case)	minimize choice frequency prediction error
$\boldsymbol{\beta}$ is alternative-constant	linear probability model
covariance $\boldsymbol{\Sigma}$ for $\boldsymbol{\varepsilon}$	MCMC
covariance $\boldsymbol{\Omega}$ for $\boldsymbol{\gamma}$	MCMC

$$\boldsymbol{U} = \boldsymbol{X}(\beta + \gamma) + \varepsilon, \quad \varepsilon \sim \mathcal{N}(0, \boldsymbol{\Sigma}),$$
$$\gamma \sim \mathcal{N}(0, \boldsymbol{\Omega})$$

$$\boldsymbol{U} = \boldsymbol{X}(\underbrace{\beta + \gamma}_{\beta_n}) + \varepsilon, \quad \varepsilon \sim \mathcal{N}(0, \boldsymbol{\Sigma}),$$

$$\beta_n \sim \mathcal{N}(\beta, \boldsymbol{\Omega})$$

$$\boldsymbol{U} = \boldsymbol{X}(\underbrace{\boldsymbol{\beta} + \boldsymbol{\gamma}}_{\boldsymbol{\beta}_n}) + \boldsymbol{\varepsilon}, \quad \boldsymbol{\varepsilon} \sim \mathcal{N}(0, \boldsymbol{\Sigma}), \quad \boldsymbol{\beta}_n \sim \mathcal{N}(\boldsymbol{\beta}, \boldsymbol{\Omega})$$

1. assume $\boldsymbol{\beta}$ is known
2. for $\approx$ 200 iterations:
   2.1 $\boldsymbol{U} \mid \boldsymbol{\Sigma}, (\boldsymbol{\beta}_n), \boldsymbol{X}, y \sim$ truncated $\mathcal{N}$
   2.2 $\boldsymbol{\beta}_n \mid \boldsymbol{\beta}, \boldsymbol{\Omega}, \boldsymbol{U} \sim \mathcal{N}$ for each n
   2.3 $\boldsymbol{\Sigma} \mid (\boldsymbol{\beta}_n), \boldsymbol{U} \sim \mathcal{W}^{-1}$
   2.4 $\boldsymbol{\Omega} \mid (\boldsymbol{\beta}_n), \boldsymbol{\beta} \sim \mathcal{W}^{-1}$
3. derive $\widehat{\boldsymbol{\Sigma}}$, $\widehat{\boldsymbol{\Omega}}$ as sample average of $\boldsymbol{\Sigma}$, $\boldsymbol{\Omega}$

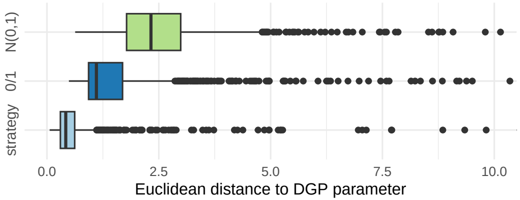
$$\boldsymbol{U} = \boldsymbol{X}(\underbrace{\beta + \gamma}_{\beta_n}) + \varepsilon, \quad \varepsilon \sim \mathcal{N}(0, \Sigma),$$
$$\beta_n \sim \mathcal{N}(\beta, \Omega)$$

1. assume β is known
2. for ≈ 200 iterations:
   2.1 $\boldsymbol{U} \mid \Sigma, (\beta_n), \boldsymbol{X}, y \sim$ truncated $\mathcal{N}$
   2.2 $\beta_n \mid \beta, \Omega, \boldsymbol{U} \sim \mathcal{N}$ for each n
   2.3 $\Sigma \mid (\beta_n), \boldsymbol{U} \sim \mathcal{W}^{-1}$
   2.4 $\Omega \mid (\beta_n), \beta \sim \mathcal{W}^{-1}$
3. derive $\widehat{\Sigma}$, $\widehat{\Omega}$ as sample average of Σ, Ω

Simulation: $\boldsymbol{\Omega} \sim \mathcal{W}^{-1}(P \cdot \boldsymbol{I}, P+1)$, $J = 3$, $T = 10$

N	100	■	500	□
P	3	■	5	□
$\boldsymbol{\Sigma}$	$\begin{pmatrix} 1 & 0 & \cdots \\ 0 & 1 & \\ \vdots & & \ddots \end{pmatrix}$	■	$\begin{pmatrix} 3 & 1 & \cdots \\ 1 & 3 & \\ \vdots & & \ddots \end{pmatrix}$	□

Average computation time: 3 seconds



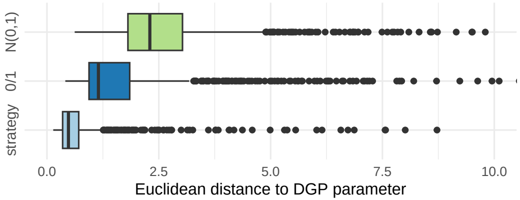
$$\boldsymbol{U} = \boldsymbol{X}(\underbrace{\beta + \gamma}_{\beta_n}) + \varepsilon, \quad \varepsilon \sim \mathcal{N}(0, \Sigma), \quad \beta_n \sim \mathcal{N}(\beta, \Omega)$$

1. assume β is known
2. for ≈ 200 iterations:
   2.1 $\boldsymbol{U} \mid \Sigma, (\beta_n), \boldsymbol{X}, y \sim$ truncated $\mathcal{N}$
   2.2 $\beta_n \mid \beta, \boldsymbol{\Omega}, \boldsymbol{U} \sim \mathcal{N}$ for each n
   2.3 $\boldsymbol{\Sigma} \mid (\beta_n), \boldsymbol{U} \sim \mathcal{W}^{-1}$
   2.4 $\boldsymbol{\Omega} \mid (\beta_n), \beta \sim \mathcal{W}^{-1}$
3. derive $\widehat{\Sigma}$, $\widehat{\Omega}$ as sample average of Σ, Ω

Simulation: $\boldsymbol{\Omega} \sim \mathcal{W}^{-1}(P \cdot \boldsymbol{I}, P+1)$, $J = 3$, $T = 10$

N	100	■	500	□
P	3	■	5	□
$\boldsymbol{\Sigma}$	$\begin{pmatrix} 1 & 0 & \cdots \\ 0 & 1 & \\ \vdots & & \ddots \end{pmatrix}$	□	$\begin{pmatrix} 3 & 1 & \cdots \\ 1 & 3 & \\ \vdots & & \ddots \end{pmatrix}$	■

Average computation time: 3 seconds



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$$\boldsymbol{U} = \boldsymbol{X}(\underbrace{\beta + \gamma}_{\beta_n}) + \varepsilon, \quad \varepsilon \sim \mathcal{N}(0, \Sigma), \quad \beta_n \sim \mathcal{N}(\beta, \Omega)$$

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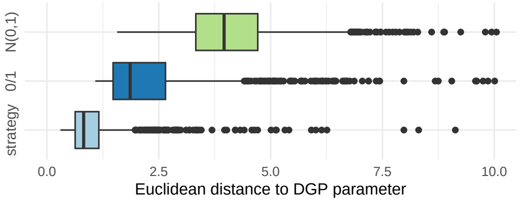
3. derive $\widehat{\Sigma}$, $\widehat{\Omega}$ as sample average of Σ, Ω

Simulation: $\boldsymbol{\Omega} \sim \mathcal{W}^{-1}(P \cdot \boldsymbol{I}, P + 1)$, $J = 3$, $T = 10$

N	100	■	500	□
P	3	□	5	■
$\boldsymbol{\Sigma}$	$\begin{pmatrix} 1 & 0 & \cdots \\ 0 & 1 & \\ \vdots & & \ddots \end{pmatrix}$	■	$\begin{pmatrix} 3 & 1 & \cdots \\ 1 & 3 & \\ \vdots & & \ddots \end{pmatrix}$	□

Average computation time: 3 seconds



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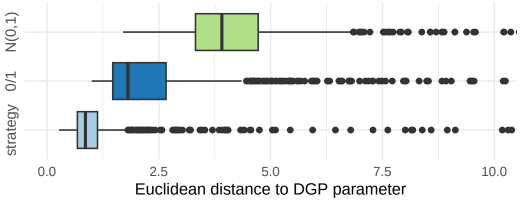
3. derive $\widehat{\Sigma}$, $\widehat{\Omega}$ as sample average of Σ, Ω

Simulation: $\boldsymbol{\Omega} \sim \mathcal{W}^{-1}(P \cdot \boldsymbol{I}, P + 1)$, $J = 3$, $T = 10$

N	100	■	500	□
P	3	□	5	■
$\boldsymbol{\Sigma}$	$\begin{pmatrix} 1 & 0 & \cdots \\ 0 & 1 & \\ \vdots & & \ddots \end{pmatrix}$	□	$\begin{pmatrix} 3 & 1 & \cdots \\ 1 & 3 & \\ \vdots & & \ddots \end{pmatrix}$	■

Average computation time: 3 seconds



$$U = X(\underbrace{\beta + \gamma}_{\beta_n}) + \varepsilon, \quad \varepsilon \sim \mathcal{N}(0, \Sigma), \quad \beta_n \sim \mathcal{N}(\beta, \Omega)$$

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2. for ≈ 200 iterations:
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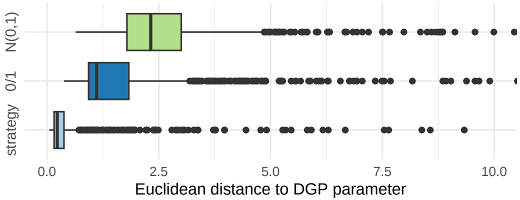
3. derive $\hat{\Sigma}$, $\hat{\Omega}$ as sample average of Σ, Ω

Simulation: $\Omega \sim \mathcal{W}^{-1}(P \cdot I, P+1)$, $J = 3$, $T = 10$

N	100	☐	500	■
P	3	■	5	☐
Σ	$\begin{pmatrix} 1 & 0 & \cdots \\ 0 & 1 & \\ \vdots & & \ddots \end{pmatrix}$	■	$\begin{pmatrix} 3 & 1 & \cdots \\ 1 & 3 & \\ \vdots & & \ddots \end{pmatrix}$	☐

Average computation time: 12 seconds



$$\boldsymbol{U} = \boldsymbol{X}(\underbrace{\beta + \gamma}_{\beta_n}) + \varepsilon, \quad \varepsilon \sim \mathcal{N}(0, \Sigma), \quad \beta_n \sim \mathcal{N}(\beta, \boldsymbol{\Omega})$$

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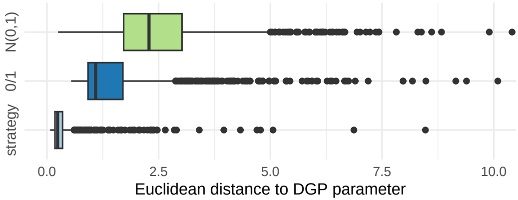
   2.3 $\Sigma \mid (\beta_n), \boldsymbol{U} \sim \mathcal{W}^{-1}$
   2.4 $\boldsymbol{\Omega} \mid (\beta_n), \beta \sim \mathcal{W}^{-1}$
3. derive $\widehat{\Sigma}$, $\widehat{\boldsymbol{\Omega}}$ as sample average of Σ, $\boldsymbol{\Omega}$

Simulation: $\boldsymbol{\Omega} \sim \mathcal{W}^{-1}(P \cdot \boldsymbol{I}, P+1)$, $J = 3$, $T = 10$

N	100	☐	500	■
P	3	■	5	☐
Σ	$\begin{pmatrix} 1 & 0 & \cdots \\ 0 & 1 & \\ \vdots & & \ddots \end{pmatrix}$	☐	$\begin{pmatrix} 3 & 1 & \cdots \\ 1 & 3 & \\ \vdots & & \ddots \end{pmatrix}$	■

Average computation time: 12 seconds



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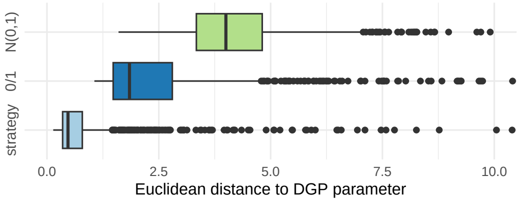
3. derive $\widehat{\Sigma}$, $\widehat{\boldsymbol{\Omega}}$ as sample average of Σ, $\boldsymbol{\Omega}$

Simulation: $\boldsymbol{\Omega} \sim \mathcal{W}^{-1}(P \cdot \boldsymbol{I}, P+1)$, $J = 3$, $T = 10$

N	100	☐	500	■
P	3	☐	5	■
Σ	$\begin{pmatrix} 1 & 0 & \cdots \\ 0 & 1 & \\ \vdots & & \ddots \end{pmatrix}$	■	$\begin{pmatrix} 3 & 1 & \cdots \\ 1 & 3 & \\ \vdots & & \ddots \end{pmatrix}$	☐

Average computation time: 12 seconds



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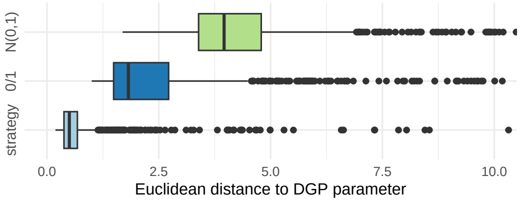
Simulation: $\boldsymbol{\Omega} \sim \mathcal{W}^{-1}(P \cdot \boldsymbol{I}, P+1)$, $J = 3$, $T = 10$

N	100	□	500	■
P	3	□	5	■
$\boldsymbol{\Sigma}$	$\begin{pmatrix} 1 & 0 & \cdots \\ 0 & 1 & \\ \vdots & & \ddots \end{pmatrix}$	□	$\begin{pmatrix} 3 & 1 & \cdots \\ 1 & 3 & \\ \vdots & & \ddots \end{pmatrix}$	■

Average computation time: 12 seconds



Outline

Putting them together

We can combine the strategies in the general "$\boldsymbol{U} = \boldsymbol{\mu} + \boldsymbol{X}(\boldsymbol{\beta} + \boldsymbol{\gamma}) + \boldsymbol{\varepsilon}$" case:

1. orthogonalize the regressors using the Frisch–Waugh–Lovell (FWL) theorem
2. get an initial estimate for $\boldsymbol{\mu}$ and $\boldsymbol{\beta}$:
   2.1 apply the presented strategies separately
   2.2 estimate a common utility scale via a linear probability model
3. perform MCMC jointly for $\boldsymbol{\Omega}$ and $\boldsymbol{\Sigma}$

Outline

Empirical application

Stated-choice experiment by Helveston et al. (2015) on car purchase decision in USA:

Attribute*	Option 1	Option 2	Option 3
Vehicle Type	Conventional 300 mile range on 1 tank	Plug-In Hybrid & 300 mile range on 1 tank (first 40 miles electric)	Electric 75 mile range on full charge
Brand	German	American	Japanese
Purchase Price	$18,000	$32,000	$24,000
Fast Charging Capability	--	Not Available	Available
Operating Cost (Equivalent Gasoline Fuel Efficiency)	19 cents per mile (20 MPG equivalent)	12 cents per mile (30 MPG equivalent)	6 cents per mile (60 MPG equivalent)
0 to 60 mph Acceleration Time**	8.5 seconds (Medium-Slow)	8.5 seconds (Medium-Slow)	7 seconds (Medium-Fast)
	○	○	X

Empirical application

Stated-choice experiment by Helveston et al. (2015) on car purchase choice (USA):

- $N = 384$ deciders
- a total of 5760 decisions
- $J = 3$ alternatives
- $P = 16$ alternative-specific regressors
- we assumed $\varepsilon \sim$ iid $\mathcal{N}(0, \sqrt{1.6})$

Empirical application

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- With random initial values, only 15% of runs converged. This took a total of 3 hours.
- Informed initial values using the presented strategy led to a single run that converged in 2 minutes to the estimates reported by Helveston et al. (2015).

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Outline

Takeaways

- RUMs are widely used in discrete choice applications, but MLE quickly becomes computational challenging (with uninformed initialization)
- We can improve optimization time and convergence rate by using consistent and numerically fast initial estimators:
  1. constant utility direction
  2. minimize choice frequency prediction error
  3. linear probability model
  4. MCMC

Takeaways

- RUMs are widely used in discrete choice applications, but MLE quickly becomes computational challenging (with uninformed initialization)

- We can improve optimization time and convergence rate by using consistent and numerically fast initial estimators:
  1. constant utility direction
  2. minimize choice frequency prediction error
  3. linear probability model
  4. MCMC

Thanks for your attention! Do you have any questions or comments?

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loelschlaeger.de/talks